**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: Option B (0.2676)

We need to find the probability that the service manager cannot meet this commitment, which means the service time exceeds 60-10=50.

First, we need to find the z-scores for 50 minutes:

Z = X-µ/σ

= 50-45/8

=5/8 => 0.625

Looking up the z-scores of 0.625 in the standard normal distribution table gives us approximately 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans: The proportion of employees older than 44 is approximately 0.15, and the proportion of employees between 38 and 44 is approximately 0.34.Since 0.34 is greater than 0.15 ,its means that there are more employees between 38 and 44 than older than 44. Therefore the statement is False.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: The above (B) statement is true.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: 2X1 ~ N(2µ , 4σ^2) and X1+X2 ~ N(2µ , 2σ^2)

From the above observations both 2X1 and X1+X2 both follows the normal distributions with the same mean(2µ) but with different variances.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: To find the values a and b that are symmetric about the mean and such that the probability of the random variable taking a value between them is 0.99

Z = X - µ/σ

µ = 100 σ = 20

We want to find z-scores that corresponds to the cumulative probability of 0.005 and 0.995 because the total probability between them is 0.99

Using standard normal distribution table we find that for a probability of 0.005 the z-score is approximately -2.58 and for 0.995 is 2.58

a = µ +(z-score\*σ)

= 100+(-2.58\*20) = 100 - 51.6 = 48.4

b = µ + (z-score\*σ)

= 100+(2.58\*20) = 100 + 51.6 = 151.6

The option D is correct answer.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: Range is Rs(99.00810 , 980.991) in Millions

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans: 5th percentile of profit (in millions) is 170.0.

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: The division 1 has the larger probability of making a loss.